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STANISŁAW LEŚNIEWSKI'S RADICAL FORMALISM

Formalism used by Leśniewski to present mathematical theory is called a radical one. According to the author, it is the implementation of the postulates of the so-called formal arithmeticians. Mathematical theory is presented as a pure game of formulas devoid of content. It is governed by the precise rules described in the metalanguage. The author stresses the difference between Leśniewski's and Hilbert's mathematical approaches.
Keywords: Stanisław Leśniewski, Foundations of Mathematics, Intuitive Formalism, Radical Formalism, Play of Inscriptions, Metamathematics.

Рассмотрен радикальный формализм Станислава Лешневского, используемый им для представления математической теории. По мнению автора, это реализация постулатов так называемых формальных арифметиков. Математическая теория представлена как чистая игра с формулами, лишенными содержания. Она регулируется точными правилами, описанными в метаязыке. Подчеркнуто различие между математическими подходами Лешневского и Гильберта.
Ключевые слова: Станислав Лешневский, основы математики, интуитивный формализм, радикальный формализм, игра надписей, метаматематика.

Розглянуто радикальний формалізм Станіслава Лешневського, що використовувався ним для представлення математиченої теорії. На думку автора, це реалізація постулатів так званих формальних арифметиків. Математична теорія представлена як чиста гра з формулами, позбавленими змісту. Вона регулюється точними правилами, описаними в метамові. Підкреслено розходження між математичними підходами Лешневського і Гільберта.
Ключові слова: Станіслав Лешневський, основи математики, інтуїтивний формалізм, радикальний формалізм, гра надписів, метаматематика.

Stanisław Leśniewski is regarded as one of the most important philosophers and logicians of the early 20th century. His views on mathematics are called „intuitive formalism": mathematics is based on the presentation of intuitive content using formal language. Both words occurring in the name of the position do not determine its equal components. The meaning of the intuitive content which the scholar should express using formal language dominates. Despite the importance of this basic – for Leśniewski – ability of mathematical cognition I do not feel equal to define it. The scholar understood it widely, he referred to various types of intuition (e. g. the sensory one, the intuition of common sense, etc.)[1].

The declared „formalism" is an important but subordinate component. Leśniewski completed its characterization with an additional term indicating the need for full elimination of the content dimension of the language of mathematics. „I advocate a rather

[1] Certainly, Leśniewski knew very well about its shortcomings revealed in Russell's famous letter. One can even look at his mature works as a way of dealing with antinomy.

radical «formalism» in construction of my system even though I am an obdurate «intuitionist»" [15, p. 487]. Intuitive mathematical theory should be formalized so that the language of mathematics in use was, in principle, deprived of any content and would become a pure play of inscriptions.

To better visualize the project of Leśniewski's formalism, I will first draw attention to the views of the so-called radical formalists, who – as I believe – formulated the pattern Leśniewski was aiming to realize. In his publications the Polish scholar never referred to the views of those scientists, but knew them as an attentive reader of *Grundgesetze der Arithmetik* [4]. For him Frege's work – as he used to emphasize – was even an unattainable pattern of accuracy[2] [14, p. 177]. In the second volume of the above mentioned work the views of Carl Johannes Thomae and Heinrich Eduard Heine are first presented and then subjected to strong criticism [4, vol. 2, § § 86–129, pp. 96–133]. Their essence is easiest to be seen in the image of mathematics pictured as a chess game, which was used by the first of the above listed formalists[3]. The work of the scholar resembles the art of converting some mathematical formulas recorded at the beginning. It is performed as described by the rules of the theory. The initial arrangement of figures corresponds to the axioms. Each subsequent thesis is a new one.

The aim of the formalism criticized by Frege was to clear the language of science of various philosophical content that used to be joined to the purely scientific statement. Scholars wanted to move the unnecessary additions beyond learning. Thanks to that they wanted to eliminate such things like the disputes over the existence or non-existence of mathematical objects. In the game of chess the pieces do not indicate anything else but themselves. They can only be assigned certain properties (e. g. the directions they move in) designated by the rules of that game. The situation is similar for mathematical symbols[4]. The scholar knows the rules of writing mathematical messages. He or she uses syntactic rules which are obligatorily applied when comparing different signs. He or she knows that the symbol „1" can be replaced with the expression „3:3" and that after the symbol of any number followed by a dot and the symbol „1" two parallel horizontal dashes and again that symbol after them can be written (e. g. „3·1=3"). The so-called mathematical knowledge does not apply to some numbers or other objects difficult to describe closely which are represented by the signs that are being used (e. g. „1"). It is just the ability to properly position the graphic signs, to present them in such a way which leads to the replacement of one configuration of inscriptions with another one.

Frege strongly rejected those views. He considered such an empirical interpretation of mathematics as naive. He strongly denied it and spoke in favour of a content approach. Although the project itself was not a complete study in its nature, criticized the authors also for not executing a proper reconstruction as they had not provided for an appropriate set of rules that are to govern such a mathematical game.

Leśniewski also disagreed with the understanding of mathematics proposed by formalists. This led, among others, to the negation of the role of the intuitive content, and yet – as he believed – is the essence of mathematics.

If intuition is accepted as the basis, then the realization of the communicability and testability standards requires some opportunity to present reasoning (exterioriza-

[2] I limit myself to a very superficial summary of their views. I leave out Frege's subtle analyses.

[3] I use the words „symbol", „sign" and „inscription" not according to the semiotic tradition. I treat them as some material objects (e. g. smears of ink on a sheet of paper) examined by a physicist.

[4] The article describes five theories based on different axioms, primary terms and rules. Ϭ5 is the most sophisticated one. I will present its essential characteristics.

tion). This essential function is fulfilled by the language. The best way for Leśniewski to communicate intuitive content was the radical method of formalization [15, p. 487], i. e. using formal language subordinated to very precisely formulated laws. The game of playing with inscriptions used by the Pole ceased to be, however, treated as mathematics and returned to the traditional position of a language game.

It was John Stuart Mill who previously encouraged in his acclaimed *A System of Logic* [19] to use inscriptions. He paid attention to the method of treating – in science (mathematics) – some visible signs as numbers and using them in a purely formal (technical) way. On the other hand, according to the traditional approach he realized that they are elements of the language and represent a certain mental process. What he wrote about the transformations of inscriptions is: „(…) each of these operations corresponds to a syllogism; represents one step of the reasoning relating not to the symbols, but to the things signified by them" [19, p. 708]. Despite formal arithmeticians, he believed that the game of inscriptions does not need to be treated as a real mathematical reasoning, but it can be looked upon as a material expression of reasoning concerning certain intuitive content. He stressed that: „(…) the language should be constructed on as mechanical principles as possible" [19, p. 707]. The best situation is when the symbols are used „without any consciousness of meaning, and with only the consciousness of using certain visible or audible marks in conformity to technical rules previously laid down" [19, p. 707]. Empty graphic signs have only conventional properties specifically assigned to them by relevant records. „There is nothing, therefore, to distract the mind from the set of mechanical operations which are to be performed upon the symbols (…)" [19, p. 708]. Let me repeat once again: Mill's remarks justify the idea of transferring the formalist mathematical game unacceptable for Leśniewski to the sphere of material language objects. In the new situation, the game of inscriptions subordinated to appropriate rules can be viewed as an expression of the transformations of some intuitive content.

The game starts with an initial arrangement of figures: a certain set of inscriptions (there can be one of them) constituting a certain system of graphic symbols. Just like the chess figures, they are not assigned any content. They are only graphics of certain shapes. They will be converted using only those operations that are expressly authorized and characterized by the rules of the game (theory directives).

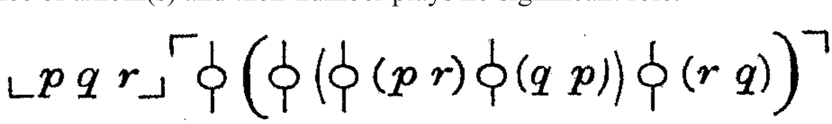
Mill's descriptions presented above basically encourage to the linguistic use of formal arithmeticians' project. Such approach, of course, was different from the point of view of the mentioned arithmeticians.

Suitability of both spheres, which Leśniewski never spoke about explicitly, provides representation to intuitive reasoning and – therefore – allows to control the intuitive process: both from the point of view of the subject as well as from the point of view of the recipient of the statement. This correspondence between mechanical operations transforming the statements as well as those mental processes were opposed by Leśniewski with the situation in which the subject goes beyond the techniques envisaged before. He performs certain operations which he justifies by referring to the intuitive content. Such extraordinary procedures that did not have a previously provided representation were described by Leśniewski as based on „intuitionistic basis of various logical secrets" [15, p. 488]. In the formalized theory they are, of course, impossible. This results in the fact that the author of the theory has it under control in some specific way: it is not only the result of his work, but it was also subjected to his conscious demands. He defined a set of initial axioms and acceptable ways of reasoning, and thus he also set the range of possible outcomes and results. The characteristics of the theory made using such mutually matched components formed the basis for distinguishing between its several versions of Leśniewski's generalized propositional calculus. I emphasize this, because – as is known – a lack of harmony between the methods of transformation that are being used and the accepted axioms is a frequent source of antinomies. According to Leśniewski, intuition is the basis of preventing them. According to him, this is the source on which the formal bases of the theories constructed by him are grounded [14, pp. 177–180].

Leśniewski took up the task of presenting mathematics as a formalized game in two important texts, i. e. in *Fundamentals of a New System of a Foundations of Mathematics* [15] published in 1929 and in *On the Foundations of Ontology* [16] presented a year later. In the first article we can find the characteristics of protothetics (the generalized propositional calculus with quantifiers) – Leśniewski's basic theory of logic, descriptions of its successive versions from Ϭ1 to Ϭ5 which have different axioms, different primitive terms and different directives that apply to them. A very accurate and detailed characterization of the formalization procedure for the richest and best developed theory Ϭ5[5] [15, § 11, pp. 467–488] occupies a very important place. The last § 12 of *Fundamentals* [15, pp. 492–605] containing an example of such a formalized theory was formed alongside the first part of the article. However, it was published only in 1938. The whole edition of this work was burnt down in the early days of World War II. Leśniewski's text was not, however, completely destroyed thanks to the fact that a few copies of the article had been preserved In the second text the formalization procedure for the next ontology (the account of names), which belongs to the system of logical theory, was presented. Because it is being developed based on protothetics extended in a certain way, Leśniewski limited himself to providing additional procedures which would allow for the previously constructed theory to be transformed into a new one, logically the later one.

Both articles mentioned above concern the problems of formalization of prothetics and ontology. Besides them, in addition, the foundations of mathematics also include mereology. Although the author of *Fundamentals* repeatedly mentioned the project of a similar approach to the last theory [15, pp. 478,488], it was never published.

Later in this article I want to briefly sketch the Polish logician's formalization method presented in *Fundamentals*. Theoretical principles of the construction of a formal language can be read from a number of theses by using which Leśniewski defined the terms necessary to formulate the directives of the theory. Each of them is the so called Terminological Explanation *(Terminologische Erklärung*, T. E.). Through a series of such statements, metalinguistic terms that specify how the theory is constructed are defined. The starting point for the construction of this metalanguage and the initial set of words of the theory of language at the same time is axiom A1[6] of the theory written on the board [15, p. 441]. In explaining the specificity of Leśniewski's formalization method the choice of axiom(s) and their number plays no significant role.

A1. 

[5] The article describes five theories based on different axioms, primary terms and rules. Ϭ5 is the most sophisticated one. I will present its essential characteristics.

[6] In traditional notation this axiom can be presented as follows (p, q, r){[(p≡r) ≡ (q≡p)] ≡ (r≡q)}.

That inscription is considered in a purely physical way as structured according to the European way of writing a string of consecutive easily identifiable meaningless graphic signs. They have specific shapes. This corresponds to the initial set of pieces on a chessboard. Generally, the scholar will deal with rearranging the pieces according to the written rules. Each subsequent arrangement will be stored. The rules of the game determine the possibility of a new configuration due to the existing systems. Corresponding directives require being acquainted with the configuration necessary to introduce a new thesis. They make use only of the physical properties of the inscriptions: their shapes and spatial relations between them. The rules (defining directives) define also the possibility to attach completely new objects used in the game.

According to Leśniewski, the protothetic game is ruled by the following directives: the directive of defining, the directive of separation of the quantifier, the directive of tearing off, the directive of substitution and the directive of extensionality.[7] These are the instructions specifying the allowable shape of the next inscription and the conditions necessary to include it as a subsequent one. As one may guess, saving the rules or instructions results in certain problems.[8] They cannot be written down in the formal language of theory. They are not, however, logical sentences. From this point of view, they cannot be expressed only through the language of a logical calculus that is being built.[9]

As I have mentioned, the formulation of directives was preceded by the construction of a metalanguage by means of which it will be possible, among others, to speak about the signs that make up the language of the theory. Usually, during his classes with the students the scholar used to devote a lot of time to these explanations (T.E.). In *Fundamentals,* in order to be precise and not take up too much space, they were formulated on the basis of symbolism used by Alfred North Whitehead and Bertrand Russell in *Principia Mathematica*.

The beginnings of construction of the metalanguage was held at the propaedeutic level. It consisted in enumerating 22 terms with intuitive meaning. They can be used in further definitions needed to formulate new directives.[10] I will not list all of them. I will present only a few basic ones [15, pp. 468–471]: „Aεb" – „A is a b"; „Id (A)" – „same object as A"; „a∩b∩…∩k" – „object which is a and b and… and k"; „vrb" – „word"; „expr" – „expression"; „prnt" – „parenthesis"; „cnf(A)" – „expression equiform to A"; „A1" – „axiom A1"; „thp" – „thesis in this system of Protothetic"; „ingr(A)" – „belonging to A"; „1ingr(A)" – „first word belonging to A".

In some cases Leśniewski would provide additional explanations clarifying their ordinary meaning. The symbols „Aεb" and „ingr (A)" are of specific nature. Their understanding was shaped primarily during Leśniewski's lectures in ontology and mereology.

I will also note down as interesting and characteristic of Leśniewski's approach the symbol „cnf (A)" [15, 470–471] presented herein. Its introduction is the result of physicalism pertaining to radical formalism. Graphic signs included in axiom A1 are only

[7] In ontology, the number of directives is higher, defining and extensionality have two types of rules.

[8] Under Lewis Carroll's influence in 1903, Russell drew attention to some problems related to the correct expression of rules [21, p. 36]. They were also indicated in Frege's analysis in 1903.

[9] In the Lvov – Warsaw School the term "spoken rules" was used. Speaking about the rules of the propositional calculus, Kotarbiński explained, „they cannot be squeezed into the symbolism of the propositional calculus" [13, p. 180].

[10] Using such propaedeutic level is somewhat similar to Frege's procedure of elucidation (*Erläuterung*, *Elucidations*) [7, p. 434]. I do not think, however, that it had to be attributed, as in Frege's, the task of characterizing something indefinable and unanalysable [3, p. 148].

some curves arising from the distribution of ink on a sheet of paper. For example, letter „p" located on the second and thirteenth place in A1 is not the same material object, but two different individuals (as they are objects occupying another place in space). They have similar shapes, but they are not identical.

The differentiation resulting from Leśniewski's physicalism is natural. Nelson Goodman acted in a very similar way by introducing the term „replica" in his work *The Structure Appearance* [6, p. 362]. Astonishment – as can easily be seen – is the result of getting used to Platonic suppositions transferred in the language of traditional mathematics. Equiform characters are identified, as they indicate the same Platonic object. For this reason, when presenting mathematical considerations in which we give up those extra ontological assumptions one has to be careful when using traditional language and the habits related to them.

Leśniewski's explanations [15, p. 471] regarding the necessity to introduce the discussed symbol are based on § 99 of Frege's considerations [4, vol. II, p. 107] over the views of radical formalists. I pay attention to it in order to identify further sources of my beliefs about the impact of the knowledge of their views on the formation of the formalistic component of the Polish scholar's position.

I will not present all subsequent T. E. With the first few explanations I will present how Leśniewski used to physicalistically characterize the initial terms. The first four T.E. describe the signs that define the beginning and end of the quantifier and the beginning and end of its range.

Terminological Explanation 1. [A]: Aεvrb.1.=.Aεcnf(1ingr(A1)) [15, p. 472].

The above-given formula describes the symbol opening the quantifier: it is called „vrb1". It is every expression equiform with the first word belonging to A1. Subsequent explanations II–IV will point to the fifth word of axiom A1 (it closes the quantifier), the sixth one (opens the quantifier's range) and the last one (closes the quantifier's range).

Explanation VII introduces the composition of objects a (Cpl(a)). It allows to describe the expressions of the theory using the term which is equivalent to the collective class of objects a, i. e. the basic category of intuitive mereology. Subsequent definitions characterize the following expressions: quantifier, variable, bracket expression, function, argument, similarity of bracket expressions, etc. In T. E. XXXIV there appears an expression of a semantic category. As can be expected, also here an approach characteristic of radical formalism will use the similarity of the shape of expressions, and it will not use the meanings which constituted the main object of Husserl's interest, which was the source of Leśniewski's idea.

Treating the expressions of a language theory as purely material objects devoid of content, which was emphasized above, does not mean giving up a repeatedly declared intuitionistic point of view. This is only some method of theory presentation (and a tool to control it). For this reason, at the end of the description of his formalisation method Leśniewski added as follows: „By no means do theories under the influence of dry and formalisation cease to consist of genuine meaningful propositions which for me are intuitively valid" [15, p. 487].

I will not carry on with the description of the introduced formalization. Its precision is commonly emphasized and attention is drawn to the very serious way of treating the procedure of defining, which is not the point of interest for the majority of authors. Leśniewski himself was also pleased with the work: his emphasized the advantage of his work over other formalizations of the basics of mathematics known to him[11] (since Frege's times until the metamathematicians from Hilbert's school) [15, p. 488], which should meet the basic requirements: consistency, restricted stringency (that allow to receive all the necessary theorems) and without any questionable interpretation. Unfortunately, he did not respond directly to Hilbert's achievements. He stressed that he would refrain from criticism until the synthesis of the results of the metamathematical school announced by the German scholar was published. It was meant to be *Grundlagen der Mathematik* written in collaboration with Paul Bernays (the first volume was published in 1934 year, the second one five years later). Despite his signals to do so, Leśniewski did not respond to it in any of his printed works.[12] He pointed to the works of Leon Chwistek [15, pp. 488–489] and John von Neumann [15, pp. 489–490] as examples of imprecise formalizations. He accused them of a significant drawback: he showed the ways in which to construct contradictions in them.

The article published in 1930 was devoted to the formalization of ontology. In paragraph 12 of *Fundamentals*[13] published a year earlier the procedure in relation to protothetics that is described above was carried out. The project of the formalization of ontology from the text of 1930 is purely potential [16, p. 608]. What was presented are: the only axiom of ontology [16, p. 609], a new set of directives and the new T.E. related to them that enable formalization. It will become a full text after the proper combination of it with the design of section 11. However, it will really become possible to implement it once section 12 has been printed.

Axiom 0. ⌊A a⌋ ⌈ ϕ (ε{A a} φ (⊢ (⌊B⌋ ⌈ ⊢ (ε{B A})⌉) ⌊B C⌋ ⌈ ϕ (φ(ε{B A} ε{C A}) ε{B C})⌉ ⌊B⌋ ⌈ ϕ (ε{B A} ε{B a})⌉))⌉ [14]

Ontology is based on protothetics. To construct it, Leśniewski's nominalist position requires adequately developed protothetics, i. e. a set of actually written out theses which were appropriately added to its directives. Among them there must be the definitions appearing in the axiom of new signs: of the thirteenth, the fifteenth and the thirty-sixth („⊢()", „φ()" „ ϕ()"), (they correspond to negation, conjunction and implication respectively).

Leśniewski also draws attention to the fact that the theses of protothetic should not contain bracktes „{", „}" present in the axiom of ontology that include arguments of the ontological „ε". [16, p. 610] They are intended for the representation of an unambiguous characterization of the category of semantic arguments.

If these requirements are met, then, according to Leśniewski, the process of building the ontology can be started [16, pp. 610–611, 625]. Write out a new axiom, adopt

[11] The formalization project that is under discussion (§ 11) was published in 1929. An example of formalization (§ 12) was delivered to the editors in the same year. However, it was printed only in 1938.

[12] The discussion over the contradiction in the system of the Hungarian formalist was very interesting. Leśniewski accused him of a vague formulation of rules and showed the possibility of building the contradictions, despite the alleged evidence showing the consistencies of a slightly weakened system [15, pp. 488–490]. Von Neumann responded in 1931 [20], introduced some changes; however, he recognized the criticism as not substantive. Leśniewski once again returned to the polemic in 1938 [17, pp. 694–707]: he accused von Neumann of erroneous understanding of the word „sign" and constructed another contradiction.

[13] I mentioned the problems with the real possibility of studying it before.

[14] Speaking informally, the axiom characterizes „ε" appearing in expression „Aεa" as follows: there is an A which is a; A is an indiviudual unit; the „ε" relationship is transitive.

the new directive and the relevant T.E. supplemented or amended respectively. Then new ontological theses can be added. Leśniewski stresses that the modified T.E. he had introduced allow to reconstruct all pre-existing theses of protothetic in the form of ontological theses. In this sense we can say that ontology includes protothetic. Leśniewski emphasizes that a small change of instruction allows to build a new set of ontological theses without the previous results being duplicated.

If you compare the T.E. presented in *Fundamentals* that are characteristic of the way of expanding the protothetic with the above given description of transition from protothetic to ontology, then – unfortunately – the latter procedure resembles the proceedings which Leśniewski described as „based on logical secrets". Inclusion of a new axiom, the change of T.E. was not anticipated in the T.E. of protothetic. While we may talk separately about the formalization of protothetic and the formalization of ontology itself within the prepared set of protothetic theses and an additional axiom, then the procedure of this change cannot be considered as formalized.

As I have mentioned, Leśniewski did not respond clearly to the views propagated as Hilbert's metamathematics. Commonly they are considered as related to modern formalism. I think that refraining from comment stems from the extremely different approach to formal language presented by both scholars. In both cases, metamathematical approach is used, but even though both scholars use this tool in a similar way, they implement it for completely different purposes.

In the conclusion to my text I will try to outline the answer to the question about the difference between Hilbert and Leśniewski in their approach to formal language. For the Polish scholar it was only a means of transferring the intuitive content. For the German scholar it became the object of the research which was to provide maths with confidence. It was not the intuitive analyses but the metamathematical research into the relationships between various formulas that was to justify its consistency. The Polish scholar considered formal expressions as material objects with which precise movements described in the game could be performed. In reality, however, they were never deprived of the intuitive content associated with them. Reasonable axioms were subjected to mechanical transformations during which no attention to their content was paid. However, the laws governing them were formulated so that you could always return to it. Therefore scholar declared as follows: „By no means do theories under the influence of such formalization cease to consist of genuinely meaningful propositions which for me are intuitively valid" [15, p. 487].

Hilbert and many other scholars disagreeing with him had a completely different attitude to the content of the signs used in axioms. If non-logical symbols are used, to which tradition ascribes certain importance, then he disregards it and believes that it is only ascribed in these axioms. In such a situation we often come across the problem of the status of this expression. It is recognized that the initial terms that occur in it denote such objects (understood broadly, e. g. individuals, classes, relations) that satisfy these axioms. Such systems of objects (there are many types of models) often exist so these terms become ambiguous in relation to the source language.[15] Sometimes treating axioms as propositions is abandoned intentionally. They are, for example, looked upon as some propositional schemes in which the so-called initial terms play the role of the variables [1, p. 191]. If a system of some broadly defined objects (e. g. individuals, classes,

[15] In fact, this is not a logical problem (because a new language is created), but a practical and communicative one.

relationships) complied with the conditions imposed on those variables in the axioms, then it also corresponded to the description included in the conclusion.

Traditionally, the meaning of axioms is built upon their authenticity. They are the statements that correctly describe a certain reality. It is possible only when understandable words are used. For this reason, the axioms do not constitute any definitions of the so called initial terms. This allowed Frege to joke about the approach proposed by Hilbert: he called his categories „pseudo-axioms" and „pseudo-propositions" [5, p. 63, 84] (propositions that did not necessarily correspond to reality and which included incomprehensible, unclear or ambiguous terms). Since axioms describe a certain reality, they have their own model. For this reason, Leśniewski, who followed into Frege's footsteps, did not have to be interested in the problems of Hilbert's metamathematics popular at that time, such as proving the non-contradiction.

Hilbert presented such an approach in 1899 in his *Grundlagen der Geometrie.* Leaving out the Euclid's method was a significant change in understanding the axiomatization of the deductive theory. The German scholar gave up assigning axioms and theorems the classically understood truthfulness and treated the theory as a hypothetical and deductive system. Hilbert's position was strongly criticized by Frege who used to think in a traditional way [5]. A famous dispute arose in which both positions were clearly defined. Leśniewski approaches the deductive scientific theory in a traditional way, close to Frege's [2]. Since axioms describe a certain reality, they have their own model. For this reason, Leśniewski, who followed into Frege's footsteps, did not have to be interested in the problems of Hilbert's metamathematics popular at that time, such as proving the non-contradiction.

In conclusion to the comparison of Leśniewski's and Hilbert's approaches to mathematical formalism, I will raise a quick point about the the problem of defining basic terms of the deductive theory. The possibility of treating axioms as definitions of initial terms protects Hilbert's approach against the allegation about the error of defining *regressus ad infinitum*. This difficulty appears in Leśniewski's traditional approach. Roman Ingarden pointed to one of its aspects [11, p. 210] when he emphasized that in fact the precision of the formal language must be based upon a colloquial language: blurry and ambiguous which together with its other flaws easily leads to contradiction. I believe Leśniewski was aware of this threat and tried to minimize it: 1) he explained the meaning of the basic terms in his propaedeutics; 2) as primitive terms occurring in the axioms he accepted those the meaning of which was the subject of lectures in intuitive mathematics; 3) he used simple unquestionable procedures of definitions (e. g. ostensive definitions); 4) if necessary, he referred to the informal methods of communication [14, pp. 374–376].

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